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ASYMPTOTIC DISTRIBUTION OF EXTREMES.(U)
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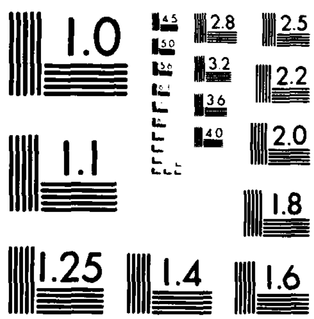

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FINAL TECHNICAL REPCRT

BY

JANOS GALAMBOS

Title: Asymptotic Distribution of Extremes

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I. RESEARCH OBJECTIVES AND ACCOMPLISHMENTS

The objective of the research program during the grant period was to clarify the applicability of the classical extreme value models to real life situations as well as to investigate dependent models when the classical models do not provide acceptable approximations to a particular problem . In applications, system reliability and material strength were to be the central focus. The domains of attraction of multivariate extreme value distributions were to be investigated.

The work has been completed with success. In addition to the objective described in the previous paragraph, the work has been supplemented by surveys on exchangeability, on the exponential distribution, and on characterizations of probability distributions. Since the 8 publications resulting from the work under this grant have been submitted to the Air Force Office of Scientific Research (see also the next section of this report), only a short summary of the results are collected here. Publication numbers refer to the list given in the next section. No additional references are given because within the publications full credit is given to eadh individual contribution to the respective field.

The following numbered paragraphs provide brief summaries and major points of the accomplishments.

1. <u>On the Classical Models</u> (applicability, characterizations, necessity of alternate models). In the classical models, the basic random variables $X_1, X_2, \ldots, X_n$ are assumed independent and identically distributed. If $F(x)$ denotes the common distribution function of the X_j, furthermore, if $Z_n = \max(X_1, X_2, \ldots, X_n)$ and $W_n = \min(X_1, X_2, \ldots, X_n)$, then $H_n(x) = P(Z_n < x) = F^n(x)$ and $L_n(x) = P(W_n < x) =$

$1 - [1 - F(x)]^n$. Hence, if $F(x)$ is known exactly, then the distribution of the extremes are readily available. However, for n 20, no approximation to $F(x)$ can lead to acceptable values for $H_n(x)$ or $L_n(x)$. Therefore , either the exact knowledge of $F(x)$ is required, which is known characterizations of distributions, or the exact formulas for $H_n(x)$ and $L_n(x)$ are to be abandoned, and their values are to be approximated by limiting forms, in which the exact form of $F(x)$ becomes irrelevant.

In practical applications, the basic assumptions of independence and stationarity (identical distributions) of the classical models are rarely, if ever, satisfied. As an approximation, however, classical models are applicable to some real life situations. Whether such an approxima - tion is justified, it can be decided by two methods. A positive answer can be obtained by developing dependent models in which the limiting properties of the extremes are identical to those in a classical model. A negative answer, on the other hand, can be obtained by some inequalities in which a limited information only is required.

Publications 1, 3 and 7 are devoted to the details of the preceding two paragraphs. Publication 1 is a major work on the described problems, except on characterizations, and it includes numerical examples for clarifying the claims and techniques. Publications 3 and 7 deal with characterizations of distribution. Although their nature is survey of the subject, these publications are the first where extreme value distributions, exponentiality, and characterizations of normality are treated by a unified approach (the method of limit laws, and the same type of functional equations for all these distributions).

<u>2. Domains of Attraction</u> (a statistical test). With the notations of the previous subsection on the classical models, a population distribution $F(x)$ is said to belong to the

domain of attraction of a possible limiting distribution $H(x)$ of a normalized maximum, if with some sequences a_n and $b_n > 0$, $H_n(a_n + b_n x) = F^n(a_n + b_n x)$ converges to $H(x)$. While analytical criteria are available to decide whether $F(x)$ belongs to the domain of attraction of $H(x)$, there were no methods of decision on such inclusion on a statistical basis. Publication 3 proposes a test of hypothesis for this purposes.

3. Material Strength. The strength S of a sheet A of metal, say, is an extreme value problem, since if X_1, $X_2, \ldots, X_n$ denote the strengths of (hypothetical) subdivisions of A, then, by the weakest link principle, S is the smallest of the X_j, $1 \leq j \leq n$. Here, S does not depend on n, hence S can be obtained as the limit of the minimum of the X_j in an appropriate dependent model. In Publication 1, starting from reasonable assumptions, a model is developed for S, in which the conclusion is that the (exact) distribution of S is Weibull.

4. System Reliability. It is a well known fact in reliability theory that the life distribution of a coherent system of components can always be representd as the maximum or the minimum of (dependent) random variables (the minimal path set or minimal cut set representation). Starting with this fact, Publication 2 gives a full account of estimates and asymptotic results for life distributions. In particular, it is shown that the approach to life distributions through asymptotic extreme value theory provides a theoretical justification for the widely used fact that the hazard rate functions of life distributions of coherent systems is u-shaped, representing a burn-in period, accidental failure period and wear-out period. An interesting consequence of the arguments of Publication 2 is that the following two assumptions, which are frequently applied in reliability theory, contradict each other: (i)a non-series coherent

system as well as its components are in the accidental failure period and (ii) the components are stochastically independent.

Notice that the accidental failure period (constant hazard rate) implies that the life distribution in this period is exponential. This fact places the exponential distribution into the family of most significant life distributions of reliability theory.

5. The Exponential Distribution. It was pointed out in the previous paragraph that the exponential distribution has a significant role in reliability theory. Publication 6 is devoted to this distribution, summarizing both its basic properties and techniques of statistical inference as they relate to exponentiality.

6. Exchangeability. An earlier result of the principal investigator is that, for extreme value problems, an arbitrary dependence can always be reduced to exchangeability. Publication 5 gives a thorough analysis of results associated with this dependence concept.

7. Multivariate Extreme Value Distributions. In recent years, much progress was made in the theory of asymptotic distributions of properly normalized componentwise extremes of multivariate data. In spite of this fact, this theory is still at its early stages. Publication 8 is a contribution to this field. A set of new criteria has been developed for multivariate distributions by which one can decide whether a distribution belongs to the domain of attraction of a particular multivariate extreme value distribution. In addition, the publication formulates a number of questions, solutions to which would be significant from the applied scientist's point of view.

II. WRITTEN PUBLICATIONS

The results of the work during the grant period have been published in the following 8 papers. 6 copies of the reprints of the first two entries and 6 copies of the preprints of entries 3-7 have been submitted to the Air Force Office of Scientific Research. Furthermore, 6 copies of entry 8 are being sent in another envelope at this time. When entries 3-7 appear, reprints (6 copies) will be sent. Entry 8 is considered to be in a Preliminary Form.

1. Extreme Value Theory in Applied Probability. The Mathematical Scientist, Vol. 6, (1981), pp. 13-26.

2. Failure Time Distributions: Estimates and Asymptotic Results. Distributions in Scientific Work (Ed.: C. Taillie et al), Vol.5 (1981), pp. 309-317 (Reidel, Dordrecht).

3. Characterizations of Distribution. Encyclopedia of Statistical Sciences Vol. 1, to appear.

4. A Statistical Test for Extreme Value Distributions. Nonparametric Methods in Statistics (Ed.:I.Vincze), to appear.

5. Exchangeability. Encyclopedia of Statistical Sciences, Vol. 2, to appear.

6. Exponential Distribution. Encyclopedia of Statistical Sciences, Vol. 2, to appear.

7. The Role of Functional Equations in Stochastic Model Building. Aequationes Mathematicae, to appear.

8. Multivariate Extreme Value Distributions. Preliminary Form, 1981.

III. INTERACTIONS

The principal investigator visited several universities in North America, Europe and Australia, and attended national and international conferences on the mentioned continents as well as in Argentina. In addition, several prominent scientists visited the principal investigator in Philadelphia. Funds from the grant, however, were spent only for domestic travel and towards partial support to deliver an invited address at the International Conference on Reliability in Karpacz, Poland (1980), and to participate in the works of the biennial meeting of the International Statistical Institute, where the principal investigator was a session organizer on extremes. The organization of this session took much time by directing and reviewing the papers of the invited speakers, which work could not have been undertaken without the support of the grant. The conference was in Buenos Aires, Argentina, 1981. Apart from the meeting in Buenos Aires, the conferences attended have been reported in the previous three Annual Technical Reports.

IV. PERSONNEL. Only the principal investigator, Janos Galambos was supported from the grant for scientific contributions. In addition, two students (Mr. Kim and Mr. Freedman) assisted the principal investigator by compiling and Xeroxing papers, and secretarial assistance was provided on an hourly basis.

V. ACKNOWLEDGEMENT . The substantial work of the principal investigator during the past four years would have been impossible without the financial support of this grant. As on several occasions in the past, the principal investigator wishes to express his appreciation for this support both to the AFOSR and personally to Dr. I.N. Shimi, Program Manager.

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Asymptotic distribution, characterizations, choice of population distribution, classical model, dependent random variables, domain of attraction, exchangeability, exponential distribution, extremes, inequalities, model building, multivariate distribution, reliability, strength of material, test of hypothesis.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This final report summarizes the results obtained by the principal investigator on the asymptotic distribution of extremes during the years 1978-1981. These results discuss the role of the choice of the population distribution in a classical model for extremes (including characterizations) as well as the necessity for deviating from the classical model for describing real life situations. Several classes of dependent random variables, with considerable detail on exchangeability, are discussed with the main focus on (CONTINUED)

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ITEM #20, CONTINUED: the behavior of their extremes. Particular emphasis is put on reliability applications and on material strength distributions. Because of the prominent role of the exponential distribution in reliability theory, the principal investigator collected material on this distribution (both on the applicability of the exponential distribution in model building and a large variety of techniques in statistical inference). In addition to asymptotic distributions, sharp inequalities are discussed which can be applied with success to two types of problems of extreme value theory: estimating system reliability and determining whether a particular assumed model contradicts reality. Extensions of some univariate results on domains of attraction to a particular asymptotic extreme value distribution are given to the multivariate case. In the univariate case, a statistical test is proposed on deciding whether the population distribution belongs to the domain of attraction of a particular extreme value distribution.

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